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BUCKLING, PREBUCKLING AND VIBRATION OF STIFFENED AND UNSTIFFENED SHELLS

Josef Singer, et al

Technion - Israel Institute of Technology

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COLLOCATION METHOD

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The stability and vibrations of stiffened and unstiffened shells and panels under axial compression are studied. The thermal buckling of shells under combined mechanical load and heating is also investigated. The studies include theoretical and experimental work, with emphasis on correlation between vibrations and buckling. Eccentricity of loading is also considered Methods for non-destructive determination of buckling loads for bars, plates and shells are developed. A new incremental theory for the nonlinear elasto-plastic behavior

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FINAL REPORT

BUCKLING, PREBUCKLING AND VIBRATION OF STIFFENED AND UNSTIFFENED SHELLS

1972, September 16 - 1974, June 30

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TAE REPORT No. 228

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SUMMARY

The stability and vibrations of stiffened and unstiffened shells and panels under axial compression are studied. The thermal buckling of shells under combined mechanical load and heating is also investigated. The studies include theoretical and experimental work, with emphasis on correlation between vibrations and buckling. Eccentricity of loading is also considered. Methods for nondestructive determination of buckling loads for bars, plates and shells are developed. A new incremental theory for the nonlinear elasto-plastic behavior of structures has been developed and is applied to some problems. The influence of initial imperfections on vibrations and stability is also studied

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INTRODUCTION

This final report describes part of a continuing investigation of stability and vibrations of stiffened and unstiffened shells under different loads and load combinations as well as heating. New methods for non-destructive determination of buckling loads for elastic bars, plates and shells are also described. A new incremental theory for the non-linear elasto-plastic behaviour of structures has been developed. The work reported was performed at the Department of Aeronautical Engineering of the Technion, Israel Institute of Technology under Grant 72-2394 (and 72-2394A), during the period September 16, 1972 to June 30, 1974. Previous studies were summarized in the Final Report of Contract F44620-71-0116: "The Buckling of Shells Under Combined Loading and Thermal Stresses" by Josef Singer and Menahem Baruch, October, 1972.

During the period September 16, 1972 to June 30, 1974, the following Scientific Reports were completed. They include also studies initiated during the previous contracts.

1. "Buckling Analysis of Elastically Constrained Conical Shells Under Hydrostatic Pressure by the Collocation Method" by A. Shalev, M. Baruch and E. Nissim, TAE Report No. 137, November 1972 (under Contracts F61052-69-L-0040, OAR F44620-71-C-0116 and Grant AFOSR 72-2394).

2. "Experimental and Theoretical Analysis of the Heat Transfer in a Cylindrical Shell Heated Along Two Opposite Generators" by Y. Frum and M. Baruch, TAE Report No. 177, March 1973 (under Grant F44620-71-C-0116 and Grant AFOSR 72-2394).

3. "The Effect of Axisymmetric Initial Imperfections in the Vibrations of Cylindrical Shells under Axial Compression" by A. Rosen and J. Singer, TAE Report No. 182, July 1973.

4. "Vibrations of Axially Loaded Stiffened Cylindrical Shells - Part I: Theoretical Analysis" - by A. Rosen and J. Singer, TAE Report No. 162, February 1974.

5. "Vibrations of Axially Loaded Stiffened Cylindrical Shells - Part II: Experimental Analysis," by A. Rosen and J. Singer. TAE Report No. 163, August 1973.

6. "Buckling of Cylindrical Shells Heated along Two Opposite Generators Combined with Axial Compression", by Y. Frum and M. Baruch, TAE Report No. 180, December 1973.

7. "Further Studies on the Effect of In-Plane Boundary Conditions on the Buckling of Stiffened Cylindrical Shells", by T. Weller, TAE Report No. 120, January 1974 (Under Contracts F61052-C-0040, F44620-71-C-0116 and Grant AFOSR-72-2394).

8. "Buckling of Discretely Stringer-Stiffened Cylindrical Shells and Elastically Restrained Panels" by Josef Singer and Raphael Haftka, TAE Report No. 91, May 1974.

9. "The Influence of Boundary Conditions on the Buckling of Stiffened Cylindrical Shells", by J. Singer and A. Rosen, TAE Report No. 213, June 1974.

10. "Vibrations and Buckling of Eccentrically Loaded Stiffened Cylindrical Shells", by A. Rosen and J. Singer, TAE Report No. 205, August 1974.

During the same period September 16, 1972 to June 30, 1974, the following papers, some of which are condensed versions of the above cited TAE Reports, were published.

1. "Integral Equations for Nondestructive Determination of Buckling Loads for Elastic Plates and Bars", by M. Baruch, Israel Journal of Technology, Vol. 11, Nos. 1-2, pp. 1-8, 1973, Proc. XV Israel Ann. Conf. Aviation on Astronautics, March 1973.

2. "Buckling Analysis of Elastically Constrained Stiffened Conical Shells under Hydrostatic Pressure by the Collocation Method" by A. Shalev, M. Baruch and E. Nissim, Israel Journal of Technology, Vol. 11, Nos. 1-2, pp. 75-87, 1973, Proc. XV Israel Ann. Conf. Aviation and Astronautics, March 1973.

3. "Buckling of Cylindrical Panels under Non-Uniform Axial Compression" by D. Durban and J. Singer, Israel Journal of Technology, Vol. 11, Nos. 1-2, pp. 9-16, 1973, Proc. XV Israel Ann. Conf. Aviation and Astronautics, March 1973.

4. "Buckling Analysis of Hydrostatically Loaded Conical Shells by the Collocation Method" by A. Shalev, M. Baruch and E. Nissim, AIAA Journal, Vol. 11, No. 12, pp. 1603-1607, December 1973, (presented at the AIAA/ASME/SAE 14th Structures, Structural Dynamics and Materials Conference held at Williamsburg, Virginia, March 20-27, 1973).

5. "Further Experimental Studies on Buckling of Integrally Ring-Stiffened Cylindrical Shells", by T. Weller and J. Singer, Experimental Mechanics, Vol. 14, No. 7, pp. 267-273, July 1974, (presented at the 3rd SESA International Congres of Experimental Mechanics, Los Angeles, May 13-18, 1973).

6. "Vibrations of Axially Loaded Stiffened Cylindrical Shells", by A. Rosen and J. Singer, Journal of Sound and Vibrations, Vol. 34, No. 3, pp. 357-378, June 1974.

7. "Floating Piecewise Linear Approximation of a Non-Linear Constitutive Equation," by D. Durban and M. Baruch, AIAA Journal, Vol. 12, No. 6, pp. 868-870, June 1974.

8. "Behaviour of an Incrementally Elastic Thick Walled Sphere under Internal and External Pressure, by D Durban and M. Baruch, to be published in the International Journal of Non-Linear Mechanics.

9. "Experimental Analysis of the Heat Transfer in a Cylindrical Shell Heated Along a Generator", by Y. Frum and M. Baruch, to be published in Journal of Experimental Mechanics.

10. "Elasto-Plastic Behaviour of a Spherical Membrane under Internal Pressure", by D. Durban and M. Baruch, Israel Journal of Technology, Vol. 12, No. 1, pp. 23-30, 1974, Proceedings of the 16th Israel Annual Conference on Aviation and Astronautics, August 1974.

11. "Influence of Eccentricity of Loading on Buckling of Stringer-Stiffened Cylindrical Shells", by T. Weller, J. Singer and S.C. Batterman, Thin Shell Structures: Theory, Experiment and Design, Y.C. Fung and E.E. Sechler, editors, Prentice Hall, pp. 305-324, 1974.

12. "Effect of Axisymmetric Imperfections on the Vibrations of Cylindrical Shells under Axial Compression", by A. Rosen and J. Singer, AIAA Journal, Vol. 12, No. 7, pp. 995-997, July 1974.

13. "Design Criteria for Buckling and Vibration of Imperfect Stiffened Shells", by J. Singer and A. Rosen, presented at the 9th Congress of the International Council of the Aeronautical Sciences, Haifa, Israel 25-30, August 1974, ICAS Paper 74-06, to be published in the proceedings.

The writers would like to take the opportunity to thank the authors whose names appear on the Scientific Reports and publications for their invaluable contributions to the work performed. They would also like to acknowledge the continuous assistance and encouragement given by the Air Force Office of Scientific Research and its European Office of Aerospace Research.

A. SUMMARY OF THEORETICAL INVESTIGATIONS

1. Panels Under Non-Uniform Axial Load

The buckling of cylindrical panels under non-uniform axial compression is investigated. Various load distributions are considered including bending and concentrated load. Along the straight edges two types of boundary conditions are examined: SS3 and SS4. Along the curved edges only the classical SS3 boundary condition is considered. The analysis is based on the Donnell equations and the standard Galerkin procedure. Results show that the critical stress depends strongly on the load distribution and on the geometric parameter K*. The critical stress increases as K* decreases and as the load distribution becomes sharper. For $K^* \to \infty$ the critical stress approaches asymptotically the value corresponding to the critical stress in uniform axial compression. No significant difference is found between the boundary conditions SS3 and SS4, the latter yielding slightly higher critical stresses.

Details are given in T.A.E. Report No. 136 and in a paper based on this report and presented at the Fifteenth Israel Annual Conference on Aviation and Astronautics, 1973, Tel Aviv, Haifa, Israel (paper No. 3 of the above listed published papers).

2. Collocation Method

a) Buckling Analysis of Elastically Constrained Stiffened Conical Shells Under Hydrostatic Pressure by the Collocation Method.

A brief review of the collocation method is presented, and an analysis by this method of an elastically constrained, stiffened conical shell subjected to hydrostatic pressure is carried out. The equations for the elastic boundary conditions of the ring and stringer-stiffened conical shells are obtained. Polynomials are then assumed as approximating functions and are substituted to yield the approximating equations. A special matrix scheme is conceived to achieve more efficient computer processing. Numerical results are obtained for a set of twelve boundary conditions (SS1W to SS4W, SS12, SS13, SS24, SS34, SS1F to SS4F) applied to a particular shell configuration. The results are presented graphically and show a typical variation of the critical load due to the stiffening of a pre-selected boundary condition. Buckling shapes resulting from the different sets of the boundary conditions are also presented. The various results obtained are discussed and some new physical conclusions are drawn.

Details are given in TAE Report No. 137 and in a paper based on this report and presented at the Fifteenth Israel Annual Conference on Aviation and Astronautics 1973, Tel-Aviv, Haifa, Israel (paper No. 2 of the above listed published papers).

b) Buckling Analysis of Elastically Constrained Non-Stiffened Conical Shells Under Hydrostatic Pressure by the Collocation Method.

A collocation method is proposed as an alternative to the conventional methods of solving buckling problems. The method is applied to hitherto unsolved buckling problems Those are elastically supported conical shells of given geometry subjected to hydrostatic pressure. Five possible elastic supports are assumed and the variations of the critical load due to change in the support stiffnesses are determined The suitability of the collocation method for buckling problems analysis is displayed in its ability to satisfy complicated boundary conditions and r ,e differential equations with continuous derivatives

Details are given in TAE Report No 137 and in a paper based on this report and presented at AIAA/ASME/SAE 14th Stru tures, Structural Dynamics and Materials Conference, Williamsbrug, Virginia, March 1973 (paper No 4 of the above listed published papers)

3. Nondestructive Determination of Buckling Loads for Elastic Plates and Bars

This subject was not included in the proposal of the present Grant, but evolved as a byproduct of the research carried out under the Grant Some new ideas have been proposed

A simple method for defining the kernel of an integral equation associated with the buckling of elastic bars and plates is proposed The plates and the bars may have varying stiffness and any combination of elastic boundary conditions and varying in-plane load. The buckling load may appear in the boundary conditions The kernel has a simple mechanical interpretation On the basis of this interpretation a nondestructive method for determination of the buckling load is proposed. By measuring the deflections and the rotations caused by a unit load, one can define the kernel experimentally avoiding the determination of the existing real boundary conditions. Having the experimentally defined kernel, one can calculate the buckling load in a nondestructive way.

Details are given in a paper presented at the Fifteenth Israel Annual Conference on Aviation and Astronautics - 1973, Tel-Aviv, Haifa, Israel (paper No. 1 of the above listed published papers).

4. Compliance with In-plane Boundary Conditions - Stiffened Cylindrical Shells

The effects of in-plane boundary conditions on the buckling loads of simply supported shells, stiffened by stringers and by rings and stringers is studied. The effects differ appreciably from those in isotropic or ring - stiffened shells and depend upon stiffener geometry and location. The axial restraint, u=0, is the predominant factor and the "weak in shear" $N_{x\phi} = 0$, boundary conditions have only a minor influence. The critical loads corresponding to the SS2 and SS4 boundary conditions are appreciably increased compared to the ones obtained for the "classical" boundary conditions and the effect is more pronounced for internal stringers. The loads corresponding to the SS1 conditions are almost identical with the classical critical loads. As the stringers become weaker, the influence of the axial restraint diminishes and the vanishing tangential constraint dominates with a tendency to approach the behaviour of an isotropic shell for very weakly stiffened shells.

For shells stiffened by rings and stringers the influence of the in-plane boundary conditions is governed by the relative magnitudes of the rings and stringers. Combinations of heavy rings and weak stringers tend to behave like isotropic shells whereas for the opposite combinations the behaviour is identical with that of stringer-stiffened shells.

Details are given in TAE Report 120. This report, discussed in the previous Contract, has been extended and includes many additional calculations.

5. Stiffened Elliptical Shells

A computer program has been prepared on the basis of a method developed for analysis of the buckling of stiffened elliptical shells under axial compression. This study will be continued.

6. Elastic-Plastic Instability of Shells

A new theory of the incremental elastic-plastic behaviour of solids is being developed. The theory is used to develop a new incremental elastic-plastic theory of beams, plates and shells. It has already been employed in some simple problems and the study will be continued:

a) Floating Piecewise Linear Approximation of a Nonlinear Constitutive Equation

One of the main dificulties arising in large deflection analysis of structures has its origin in the non-linearity of the stress-strain relations. This difficulty is usually overcome by piecewise linearization of the constitutive equation. A method for construction of the "best" piecewise linear approximation is proposed. It is shown, through an examination of a simple model, that the suggested method yields a sequence of upper bounds to the load-deflection behaviour. The maximum load carrying capacities, for the given model, obtained by the approximate and the exact methods are the same.

It is shown also that for the simple model considered, the envelope of the floating piecewise linear approximation represents the exact solution.

Details are given in a paper published in the AIAA Journal (Paper No. 7 of the above listed published papers).

b) Behaviour of Incrementally Elastic Thick Walled Sphere under Internal and External Pressure.

The behaviour of a thick walled sphere under internal and external pressure is considered. The material of the sphere is assumed to obey an incrementally elastic constitutive law. There is no restriction on the size of the deformation and a solution is given in terms of special functions associated with the non-linear differential equations of the problem.

As a numerical example, the behaviour of a spherical shell subjected to internal pressure is described. It is shown that at a certain critical pressure instability of the second kind (inflation) is obtained.

Details are given in a paper to be published in the International Journal of Non-Linear Mechanics (paper No. 8 of the above listed papers).

c. Elasto-Plastic Behaviour of a Thin Walled Spherical Shells under Internal Pressure.

An incremental approach is applied to define the elasto-plastic behaviour of a thin spherical shell under internal pressure. Material and geometrical non-linearities are taken into account. A general solution is given. For several special cases the inflation points are obtained and compared with existing solutions. In addition, a simple method for defining the upper bound behaviour of an elasto-plastic structure, proposed earlier for a simple model, is applied and demonstrated for a more general case.

Details are given in a paper presented at the Sixteenth Annual Conference on Aviation and Astronautics, Tel-Aviv, August 1974 (paper No. 10 of the above listed papers).

7. Thermal Buckling

The combination of mechanical and thermal stresses may cause the failure of shells by instability. This combination cannot be checked and analysed without taking into consideration the heat transfer from the heat source to the shell, into shell and out of it. A closed form theoretical solution for the heat transfer of a cylindrical shell heated at two opposite generators, has been developed.

Details are given in TAE 177 and in a paper to be published in the Journal of Experimental Mechanics (paper No. 9 of the above listed papers).

The non-linear analysis for thermal buckling of cylindrical shells has been continued. The development of a computer program for solution by the Galerkin method of the non-linear theory for thermal loading and buckling is in progress. A theoretical study of the heat transfer in a cylindrical shell heated along a generator with finite width has also been initiated.

8. Vibrations of Axially Compressed Cylindrical Shells

Theoretical and experimental methods for investigation of the vibrations of loaded cylindrical shells were developed.

In the theoretical study, a set of equilibrium equations and boundary conditions is obtained by a variational approach for isotropic and stiffened cylindrical shells. The stiffeners are treated in the manner developed earlier for the buckling analyses of stiffened shells Three different sets were developed, one according to Donnell theory, another according to the more exact Fluegge theory and an intermediate one The equilibrium equations and boundary conditions are solved by a method known as the "exact" method, and a computer program was developed. This program is also capable of finding the buckling load, defined as the lowest load where one of the frequencies vanishes, with the vibration mode at this frequency becoming the buckling mode. This method of solution conveniently admits any desired combination of boundary conditions. The results obtained were compared with the results obtained by other investigators using different methods The agreement was found to be very good. For the many cases calculated the differences between the three sets of equations was very small.

Details are given in TAE 162 and in a paper published in the Journal of Sound and Vibrations (paper No. 6 of the above listed papers).

9. Vibrations and Buckling of Axially Compressed Stiffened Cylindrical Shells with Elastic Restraints.

In practice the boundary conditions are not any combination of the well known SS or C cases, but an intermediate case which can be represented by springs at the boundaries. At each boundary of the cylindrical shell one assumes linear springs in the axial, circumferential and radial directions as well as rotational springs. When the stiffness of the springs becomes zero or infinity, the classical types of boundary conditions are obtained. The equiblibrium equations and boundary conditions are solved by the same "exact" method which is used for the calculations described in the previous section. The influence of axial and rotational restraints on the vibrations and buckling of stiffened cylindrical shells is studied for intermediate states between SS3 and SS4 and between SS4 and C4 boundary conditions. For buckling, the earlier observed large influence of the B.C.'s on the buckling loads and modes was reconfirmed. For vibrations of stiffened shells, the influence of axial restraint was not known and is found to be large. The similar influence of boundary conditions on buckling and vibrations indicates that correlation with vibration tests may be an appropriate tool for a more precise definition of boundary conditions of stiffened shells, to be discussed in Section B. 2.

Details are given in TAE 213 and in the ICAS paper (paper No. 13 of the above listed papers).

10. Influence of In-plane Boundary Conditions and Pre-buckling Deformation on Buckling of Shells.

The influence of in-plane boundary conditions on the buckling of stiffened cylindrical shells were investigated thoroughly in TAE 120. This investigation has been extended to include also the effect of rotational restraint. In addition to widening the range of the parametric study, it is also shown that rotational restraint has an appreciable effect on the relative influence of the in-plane boundary conditions.

The influence of prebuckling deformations is studied by comparison of the results obtained with the nonlinear BOSOR 3 program with those obtained with the linear programs. The comparison is carried out for SS3, SS4 and C4 boundary conditions. Different shell and stringer geometries are studied and compared with the results of Yamaki's study for isotropic shells. The reduction in buckling load due to the buckling deformations in stiffened shells is found to be always smaller than in isotropic shells.It is found that while for relatively long shells the influence of prebuckling deformations is negligible for the cases studied, consideration of prebuckling yields much higher buckling loads in short shells.

Details are given in TAE 213 and in the ICAS paper (paper No. 13 of the above listed papers).

11. Influence of Eccentricity of Loading on the Vibrations and Buckling of Stringer Stiffened Cylindrical Shells.

The eccentricity of loading has been shown in parametric studies carried out under previous contracts to have a large influence on the buckling of stringer stiffened cylindrical shells. The investigation is now continued for other types of shells with different boundary conditions and extended to study the influence of eccentricity of loading on the vibrations of the loaded shells. It is found that eccentricity of loading has also a noticeable influence on the vibrations. Taking into account also the different point of support at the boundary (and not only the end moment) causes large variations for the SS4 case, even at zero load.

For zero eccentricity, the influence of prebuckling deformations on the vibrations is also investigated by comparison of the results obtained with the BOSOR 3 program with those of linear theory. For the cases studied, only very small differences are found between the two theories up to very high loads.

Details are given in TAE 205.

12 Influence of Initial Geometrical Imperfections on the Vibrations of Cylindrical Shells.

The influence of initial geometrical imperfections on the buckling of cylindrical shells is well known. The influence of such imperfections on the vibration has now been examined. A dynamical extension of Koiter's well known

1963 study for axisymmetric imperfections is carried out to assess the influence of the same initial imperfections on the vibrations of axially loaded shells. It is found that such imperfections have a large influence on the vibrations, even at zero load. These results agree with the results of a similar study carried out in the USSR. The present study is only a beginning, because the imperfection and vibration modes examined are not practical. Further studies are in progress on the influence of more realistic initial geometrical imperfections on realistic modes of vibration. Preliminary results show considerable influence.

Hence there appears to be a possibility to investigate the initial geometrical imperfections of shells by vibrating them and measuring their response at low loads.

B. SUMMARY OF EXPERIMENTAL INVESTIGATION

1. Vibrations of Axially Compressed Cylindrical Shells

In the experimental study which was carried out concurrently with the theoretical studies described in Section A.8, aluminum alloy shells similar to those used in the earlier buckling test are employed. The shells are vibrated by an acoustical driver positioned inside the shells and the shell response is detected by an outside microphone. When resonance is detected by Lissajous figures, the mode of vibration is obtained by moving the microphone in circumferential and axial directions around the shell while plotting the microphone reading as function of position on XY recorders. The shell is loaded by a screw-jack, and at every load the resonances for a wide range of frequencies are examined. In the first test series seven shells were tested, one isotropic, 4 stringer-stiffened and 2 ring-stiffened, all the specimens with clamped boundary conditions. The experimental results show clear trends and agree with the theoretical predictions.Different behaviour between stringer- and ring-stiffened shells is observed and confirms corresponding theoretical predictions. Experiments are continued until buckling occurs. The possibility of non-destructive determination of the buckling load by vibration was examined yielding encouraging preliminary results.

Details are given in TAE 163 and in a paper published in the Journal of Sound and Vibration (paper No. 6 of the above listed papers).

2. Experimental Definition of Boundary Conditions by Correlation with Vibration tests.

In order to examine the possibility of the precise definition of boundary conditions by vibrations, the theoretical study of Section A9 is followed by experimental work. Here the vibrations of simply supported and clamped shells are measured at different loads. The simply supported boundary conditions are assumed to be axially restrained and the clamped boundary conditions are represented by some large rotational springs at the boundaries.The experimental results are compared with theory for the vibrations and thus the magnitude of restraints are determined. Then the buckling load of the shell is calculated for these elastic restraints. 18 stringer stiffened shells with different boundary conditions, geometry and stiffeners were tested with this procedure. With this method the scatter in the ratio of experimental to predicted buckling loads is reduced from more than 70% to less than 30%.

Details are given in TAE 213 and in the ICAS paper (paper No. 13 of the above listed papers).

3. Influence of Eccentricity of Loading on the Vibrations and Buckling of Stringer-Stiffened Cylindrical Shells

Tests are carried out on two families of stringer-stiffened shells, one of 3 shells with heavy stringers and one of 3 shells with medium stringers. Each family has one shell with zero load eccentricity, one with a large outside eccentricity and one with a moderate one. Results for vibrations and buckling are compared with calculations outlined in Section All. Fair agreement was obtained.

Details are given in TAE 205, TAE 213 and the ICAS paper (paper No. 13 of the above listed papers).

4. Nondestructive Test Methods for Prediction of Buckling Loads for Stiffened Shells

Instead of curve fitting of the experimental results of the frequency squared versus the axial load used earlier, an indirect curve fitting technique has been developed - fitting straight lines to higher powers of the measured frequencies. The extrapolated intersections of these straight lines are possible predicted buckling loads. Preliminary results of this technique applied to a number of shells tested are encouraging. Study of this technique continues with the aim of providing a basis for a reliable nondestructive test method.

Details are given in TAE 213 and in the ICAS paper (paper No. 13 of th. above listed papers).

5. Thermal Buckling

a) Experimental Analysis of the Heat Transfer in a Cylindrical Shell Heated Along a Generator.

An experimental program was conducted to determine the temperature profile in a cylindrical shell heated along two opposite generators. Many tests were performed. Aluminum cylindrical shells were tested on a test rig designed for this loading case. The heating was affected by two line heaters and the temperature distribution as a function of time and space was measured by a set of thermocouples located on the surface of the shell. The agreement between the closed form solution of the respective heat transfer problem and the experiments was found to be very good.

Details are given in TAE . 177 and in a paper to be published in the Journal of Experimental Mechanics (paper No. 9 of the above listed papers).

b) Buckling of Cylindrical Shells Heated along Two Opposite Generators Combined with Axial Compression

Buckling of cylindrical shells due to rapid heating along two opposite generators and buckling due to the combination of axial load and heating was investigated. Many tests on aluminum cylindrical shells were performed. A semi-empirical correlation function for this kind of combined loading has been found.

Details are given in TAE 180. An extension of the test rig to permit tests on buckling of cylindrical shells due to combination of torque, axial load and heating along two generators has been designed.

6. Study of the Growth of Initial Imperfections of Axially Compressed Stiffened Cylindrical Shells.

Work on the semi-automatic device for measurement of initial imperfections and the growth of these imperfections during axial compression has been continued. The test apparatus is now operational. The rig employs a spiral scanning path in order to obtain a continuous record of the imperfections. A method of data acquisition by automatic sampling and feeding to the Beckman Data Acquisition system has been developed. A computer program which analyses the experimental data has been prepared. Preliminary tests are in progress.

INVENTION STATEMENT

No inventions were conceived or made during the period 16 September 1972 - 30 June 1974.